Modelling Conditional Knowledge Discovery and Belief Revision by Abstract State Machines

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Abstract. We develop a high-level ASM specification for the Condor system that provides powerful methods and tools for managing knowledge represented by conditionals. Thereby, we are able to elaborate crucial interdependencies between different aspects of knowledge representation, knowledge discovery, and belief revision. Moreover, this specification provides the basis for a stepwise refinement development process of the Condor system based on the ASM methodology.

1 Introduction

Commonsense and expert knowledge is most generally expressed by rules, connecting a precondition and a conclusion by an *if-then*-construction. If-then-rules are more formally denoted as conditionals, and often they occur in the form of probabilistic (quantitative) conditionals like "*Students are young with a probability of (about) 80 %*" and "*Singles (i.e. unmarried people) are young with a probability of (about) 70 %*", where this commonsense knowledge can be expressed formally by $\{(young|student)[0.8], (young|single)[0.7]\}$. In another setting, qualitative conditionals like $(expensive|Mercedes)[n]$ are considered where $n \in \mathbb{N}$ indicates a degree of plausibility for the conditional "*Given that the car is a Mercedes, it is expensive*".

The crucial point with conditionals is that they carry generic knowledge which can be applied to different situations. This makes them most interesting objects in Artificial Intelligence, in theoretical as well as in practical respect. Within the Condor project (Conditionals - discovery and revision), we develop methods and tools for discovery and revision of knowledge expressed by conditionals. Our aim is to design, specify, and develop the ambitious Condor system using Abstract State Machines, based on previous experiences with the ASM approach and using tools provided by the ASM community.

Figure 1 provides a bird's-eye view of the Condor system. Condor can be seen as an agent being able to take rules, evidence, queries, etc., from the environment and giving back sentences he believes to be true with a degree of

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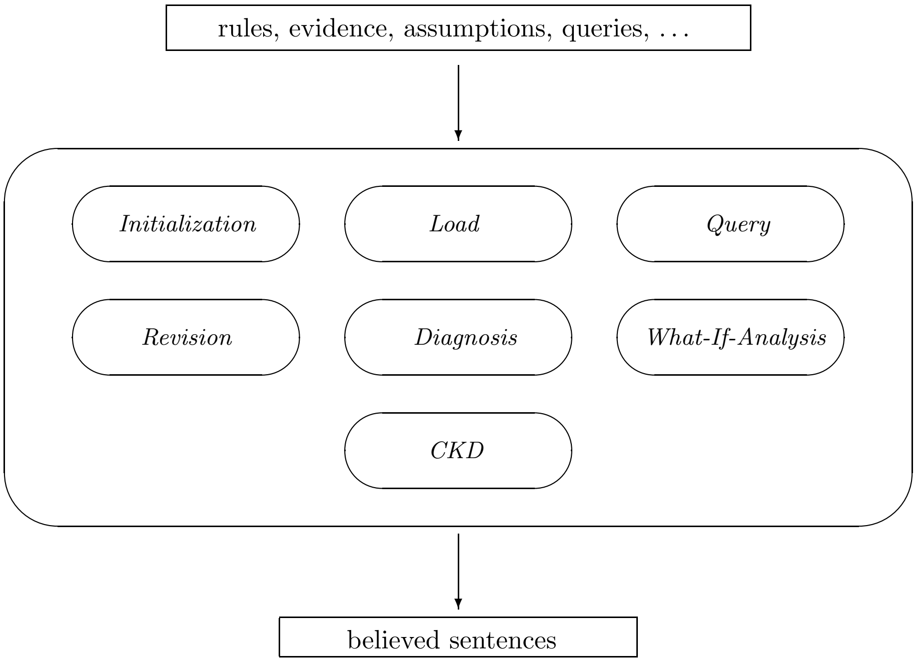

Fig. 1. A bird's-eye view of the CONDOR-Systems

certainty. Basically, these degrees of belief are calculated from the agent's current epistemic state which is a representation of his cognitive state at the given time. The agent is supposed to live in a dynamic environment, so he has to adapt his epistemic state constantly to changes in the surrounding world and to react adequately to new demands.

In this paper, we develop a basic ASM, denoted by CONDORASM, providing the top-level functionalities of the CONDOR system as they are indicated by the buttons in Figure 1. Using this ASM, we are not only able to decribe precisely the common functionalities for dealing with both quantitative and qualitative approaches. We also work out crucial interdependencies between e.g. inductive knowledge representation, knowledge discovery, and belief revision in a conditional setting. Moreover, CONDORASM provides the basis for a stepwise refinement development process of the CONDOR system.

The rest of this paper is organized as follows: In Section 2, we provide a very brief introduction to qualitative and quantitative logics. In Section 3, the universes of CONDORASM and its overall structure are introduced, while in Section 4 its top-level functions are specified. Section 5 contains some conclusions and points out further work. In Appendix A, we summarize the universes, functions, constraints, and transition rules for CONDORASM that are developed throughout this paper.

2 Background: Qualitative and Quantitative Logic in a Nutshell

We start with a propositional language $\mathcal{L}$, generated by a finite set Σ of atoms $a, b, c, \ldots$. The formulas of $\mathcal{L}$ will be denoted by uppercase roman letters $A, B, C, \ldots$. For conciseness of notation, we will omit the logical *and*-connector, writing AB instead of $A \wedge B$, and barring formulas will indicate negation, i.e. $\overline{A}$ means $\neg A$. Let Ω denote the set of possible worlds over $\mathcal{L}$; Ω will be taken here simply as the set of all propositional interpretations over $\mathcal{L}$ and can be identified with the set of all complete conjunctions over Σ. $\omega \models A$ means that the propositional formula $A \in \mathcal{L}$ holds in the possible world $\omega \in \Omega$.

By introducing a new binary operator $|$, we obtain the set $(\mathcal{L} \mid \mathcal{L}) = \{(B|A) \mid A, B \in \mathcal{L}\}$ of conditionals over $\mathcal{L}$. $(B|A)$ formalizes "*if* A *then* B" and establishes a plausible, probable, possible etc connection between the *antecedent* A and the *consequent* B. Here, conditionals are supposed not to be nested, that is, antecedent and consequent of a conditional will be propositional formulas.

To give appropriate semantics to conditionals, they are usually considered within richer structures such as *epistemic states*. Besides certain (logical) knowledge, epistemic states also allow the representation of preferences, beliefs, assumptions etc of an intelligent agent. Basically, an epistemic state allows one to compare formulas or worlds with respect to plausibility, possibility, necessity, probability etc.

Well-known qualitative, ordinal approaches to represent epistemic states are Spohn's *ordinal conditional functions, OCFs*, (also called *ranking functions*) [Spo88], and *possibility distributions* [BDP92], assigning degrees of plausibility, or of possibility, respectively, to formulas and possible worlds. In such qualitative frameworks, a conditional $(B|A)$ is valid (or *accepted*), if its confirmation, AB, is more plausible, possible etc. than its refutation, $A\overline{B}$; a suitable degree of acceptance is calculated from the degrees associated with AB and $A\overline{B}$.

In a quantitative framework, most appreciated representations of epistemic states are provided by *probability functions* (or *probability distributions*) $P : \Omega \to [0, 1]$ with $\sum_{\omega \in \Omega} P(\omega) = 1$. The probability of a formula $A \in \mathcal{L}$ is given by $P(A) = \sum_{\omega \models A} P(\omega)$, and the probability of a conditional $(B|A) \in (\mathcal{L} \mid \mathcal{L})$ with $P(A) > 0$ is defined as $P(B|A) = \dfrac{P(AB)}{P(A)}$, the corresponding conditional probability. Note that, since $\mathcal{L}$ is finitely generated, Ω is finite, too, and we only need additivity instead of σ-additivity.

3 The formal framework of CondorASM

3.1 Universes

On a first and still very abstract level we do not distinguish between qualitative and quantitative conditionals. Therefore, we use Q as the universe of qualitative and quantitative scales.

The universe Σ of propositional variables provides a vocabulary for denoting simple facts. The universe Ω contains all possible worlds that can be distinguished using Σ. $Fact_{\mathcal{U}}$ is the set of all (unquantified) propositional sentences over Σ, i.e. $Fact_{\mathcal{U}}$ consists of all formulas from $\mathcal{L}$. The set of all (unquantified) conditional sentences from $(\mathcal{L} \mid \mathcal{L})$ is denoted by $Rule_{\mathcal{U}}$.

The universe of all sentences without any qualitative or quantitative measure is given by

$$Sen_{\mathcal{U}} = Fact_{\mathcal{U}} \cup Rule_{\mathcal{U}}$$

with elements written as A and $(B|A)$, respectively. Additionally, $SimpleFact_{\mathcal{U}}$ denotes the set of simple facts $\Sigma \subseteq Fact_{\mathcal{U}}$, i.e. $SimpleFact_{\mathcal{U}} = \Sigma$.

Analogously, in order to take quantifications of belief into account, we introduce the universe $Sen_{\mathcal{Q}}$ of all qualitative or quantitative sentences by setting

$$Sen_{\mathcal{Q}} = Fact_{\mathcal{Q}} \cup Rule_{\mathcal{Q}}$$

whose elements are written as $A[x]$ and $(B|A)[x]$, respectively, where $A, B \in Fact_{\mathcal{U}}$ and $x \in Q$. For instance, the measured conditional $(B|A)[x]$ has the reading *if A then B with degree of belief x*. The set of measured simple facts is denoted by $SimpleFact_{\mathcal{Q}} \subseteq Fact_{\mathcal{Q}}$.

The universe of epistemic states is given by $EpState \subseteq \{\Psi \mid \Psi : \Omega \to Q\}$. We assume that each $\Psi \in EpState$ uniquely determines a function (also denoted by Ψ) $\Psi : Sen_{\mathcal{U}} \to Q$. For instance, in a probabilistic setting, for $\Psi = P : \Omega \to [0,1]$ we have $P(A) = \sum_{\omega \models A} P(\omega)$ for any unquantified sentence $A \in Sen_{\mathcal{U}}$.

Finally, there is a binary satisfaction relation (modelled by its characteristic function in the ASM framework) $\models_{\mathcal{Q}} \subseteq EpState \times Sen_{\mathcal{Q}}$ such that $\Psi \models_{\mathcal{Q}} S$ means that the state Ψ satisfies the sentence S. Typically, Ψ will satisfy a sentence like $A[x]$ if Ψ assigns to A the degree x ("*in Ψ, A has degree of probability / plausibility x*").

In this paper, our standard examples for epistemic states are probability distributions, but note that the complete approach carries over directly to the ordinal framework (see eg. [KI01a]).

Example 1. In a probabilistic setting, conditionals are interpreted via conditional probability. So for a probability distribution P, we have $P \models_{\mathcal{Q}} (B|A)[x]$ iff $P(B|A) = x$ (for $x \in [0,1]$).

3.2 Overall structure

In the CondorASM, the agent's current epistemic state is denoted by the controlled nullary function[1]

$$currstate : EpState$$

The agents beliefs returned to the environment can be observed via the controlled function

$$believed\_sentences : \mathcal{P}(Sen_{\mathcal{Q}})$$

[1] For a general introdcution to ASMs and also to stepwise refinement using ASMs see e.g. [Gur95] and [SSB01]; in particular, we will use the classification of ASM functions – e.g. into controlled or monitored functions – as given in [SSB01].

input type		monitored nullary function
$\mathcal{P}(Sen_{\mathcal{Q}})$	:	*rule_base* *new_information* *assumptions*
$\mathcal{P}(Sen_{\mathcal{U}})$	:	*queries* *goals*
$\mathcal{P}(Fact_{\mathcal{Q}})$	:	*evidence*
$\mathcal{P}(SimpleFact_{\mathcal{U}})$	:	*diagnoses*
EpState	:	*stored_state* *distribution*
RevisionOp	:	*rev_op*

Fig. 2. Monitored function in CondorASM

with $\mathcal{P}(S)$ denoting the power set of S.

As indicated in Figure 1, there are seven top-level functions that can be invoked, ranging from initialization of the system to the automatic discovery of conditional knowledge (CKD). Thus, we have a universe

$$\begin{aligned} \mathit{WhatToDo} \ = \{ & \mathit{Initialization}, \mathit{Load}, \mathit{Query}, \mathit{Revision}, \\ & \mathit{Diagnosis}, \mathit{What\text{-}If\text{-}Analysis}, \mathit{CKD} \} \end{aligned}$$

The nullary interaction function

$$do : \mathit{WhatToDo}$$

is set by the environment in order to invoke a particular function and is reset by CondorASM on executing it.

The appropriate inputs to the top-level functions are modelled by monitored nullary functions set by the environment. For instance, simply querying the system takes a set of (unquantified) sentences from $Sen_{\mathcal{U}}$, asking for the degree of belief for them. Similarly, the *What-If-Analysis* realizes hypothetical reasoning, taking a set of (quantified) sentences from $Sen_{\mathcal{Q}}$ as assumptions, together with a set of (unquantified) sentences from $Sen_{\mathcal{U}}$ as goals, asking for the degree of belief for these goals under the given assumptions. Figure 2 summarizes all monitored functions serving as inputs to the system; their specific usage will be explained in detail in the following section along with the corresponding top-level functionalities.

4 Top-level Functions in the Condor-System

4.1 Initialization

In the beginning, a prior epistemic state has to be built up on the basis of which the agent can start his computations. If no knowledge at all is at hand, simply the uniform epistemic state, modelled by the nullary function

$$uniform : EpState$$

is taken to initialize the system. For instance, in a probabilistic setting, this corresponds to the uniform distribution where everything holds with probability 0.5.

If, however, default knowledge or a set of probabilistic rules is at hand to describe the problem area under consideration, an epistemic state has to be found to appropriately represent this prior knowledge. To this end, we assume an inductive representation method to establish the desired connection between sets of sentences and epistemic states. Whereas generally, a set of sentences allows a (possibly large) set of models (or epistemic states), in an inductive formalism we have a function

$$inductive : \mathcal{P}(Sen_{\mathcal{Q}}) \rightarrow EpState$$

such that $inductive(S)$ selects a unique, "best" epistemic state from all those states satisfying S. Starting with the no-knowledge representing state *uniform* can be modelled by providing the empty set of rules since the constraint

$$uniform = inductive(\emptyset)$$

must hold.

Thus, we can initialize the system with an epistemic state by providing a set of (quantified) sentences S and generating a full epistemic state from it by inductively completing the knowledge given by S. For reading in such a set S, the monitored nullary function $rule\_base : \mathcal{P}(Sen_{\mathcal{Q}})$ is used:

if $do = Initialization$
  then $currstate := inductive(rule\_base)$
    $do := undef$

Selecting a "best" epistemic state from all all those states satisfying a set of sentences S is an instance of a general problem which we call the *representation problem* (cf. [BKI02a]). There are several well-known methods to model such an inductive formalism, a prominent one being the *maximum entropy* approach.

Example 2. In a probabilistic framework, the *principle of maximum entropy* associates to a set $\mathcal{S}$ of probabilistic conditionals the unique distribution $P^* = MaxEnt(\mathcal{S})$ that satisfies all conditionals in $\mathcal{S}$ and has maximal entropy, i.e., $MaxEnt(\mathcal{S})$ is the (unique) solution to the maximization problem

$$\max H(P') = -\sum_{\omega} P'(\omega) \log P'(\omega) \quad (1)$$
$$\text{s.t. } P' \text{ is a probability distribution with } P' \models \mathcal{S}.$$

The rationale behind this is that $MaxEnt(\mathcal{S})$ represents the knowledge given by $\mathcal{S}$ most faithfully, i.e. without adding information unnecessarily (cf. [Par94,PV97,KI98]).

We will illustrate the maximum entropy method by a small example.

Example 3. Consider the three propositional variables a - *being a student*, b - *being young*, and c - *being unmarried. Students* and *unmarried people* are mostly *young*. This commonsense knowledge an agent may have can be expressed probabilistically e.g. by the set $\mathcal{S} = \{(b|a)[0.8], (b|c)[0.7]\}$ of conditionals. The *MaxEnt*-representation $P^* = \mathit{MaxEnt}(\mathcal{S})$ is given in the following table[2]:

ω	$P^*(\omega)$	ω	$P^*(\omega)$	ω	$P^*(\omega)$	ω	$P^*(\omega)$
abc	0.1950	$ab\overline{c}$	0.1758	$a\overline{b}c$	0.0408	$a\overline{b}\overline{c}$	0.0519
$\overline{a}bc$	0.1528	$\overline{a}b\overline{c}$	0.1378	$\overline{a}\overline{b}c$	0.1081	$\overline{a}\overline{b}\overline{c}$	0.1378

4.2 Loading an epistemic state

Another way to initialize the system with an epistemic state is to load such a state directly from the environment (where it might have been stored during a previous run of the system; this could be modelled easily by an additional top-level function). Therefore, there is a monitored nullary function *stored_state* : *EpState* which is used in the following rule:

if *do* = *Load*
  then *currstate* := *stored_state*
    do := *undef*

4.3 Querying an epistemic state

The function

$$\mathit{belief}\colon\ \mathit{EpState} \times \mathcal{P}(\mathit{Sen}_{\mathcal{U}}) \to \mathcal{P}(\mathit{Sen}_{\mathcal{Q}})$$

is the so-called belief measure function which is subject to the condition

$$\mathit{belief}(\Psi, \mathcal{S}) \ = \ \{S[x] \mid S \in \mathcal{S} \text{ and } \Psi \models_{\mathcal{Q}} S[x]\}$$

for every $\Psi \in \mathit{EpState}$ and $S \subseteq \mathit{Sen}_{\mathcal{U}}$. For a given state Ψ, the call $\mathit{belief}(\Psi, S)$ returns, in the form of measured sentences, the beliefs that hold with regard to the set of basic sentences $S \subseteq \mathit{Sen}_{\mathcal{U}}$. The monitored function $\mathit{queries} : \mathcal{P}(\mathit{Sen}_{\mathcal{U}})$ holds the set of sentences and is used in the rule:

if *do* = *Query*
  then *believed_sentences* := *belief*(*currstate*, *queries*)
    do := *undef*

Example 4. Suppose the current epistemic state is $\mathit{currstate} = \mathit{MaxEnt}(\mathcal{S})$ from Example 3 above, and our query is "*What is the probability that unmarried students are young?*", i.e. $\mathit{queries} = \{(b|ac)\}$. The system returns $\mathit{belief}(\mathit{currstate}, \mathit{queries}) = \{(b|ac)[0.8270]\}$, that is, unmarried students are supposed to be young with probability 0.8270.

[2] *MaxEnt*($\mathcal{S}$) has been computed with the expert system shell SPIRIT [RKI97a,RKI97b]; cf. `http://www.fernuni-hagen.de/BWLOR/spirit.html`

4.4 Revision of Conditional Knowledge

Belief revision, the theory of dynamics of knowledge, has been mainly concerned with propositional beliefs for a long time. The most basic approach here is the *AGM-theory* presented in the seminal paper [AGM85] as a set of postulates outlining appropriate revision mechanisms in a propositional logical environment. This framework has been widened by Darwiche and Pearl [DP97a] for (qualitative) epistemic states and conditional beliefs. An even more general approach, unifying revision methods for quantitative and qualitative representations of epistemic states, is described in [KI01a]. The crucial meaning of conditionals as *revision policies* for belief revision processes is made clear by the so-called *Ramsey test* [Ram50], according to which a conditional $(B|A)$ is accepted in an epistemic state Ψ, iff revising Ψ by A yields belief in B:

$$\Psi \models (B|A) \quad \text{iff} \quad \Psi * A \models B \tag{2}$$

where $*$ is a belief revision operator (see e.g. [Ram50,BG93]).

Note, that the term "belief revision" is a bit ambiguous: On the one hand, it is used to denote quite generally *any* process of changing beliefs due to incoming new information [Gär88]. On a more sophisticated level, however, one distinguishes between different kinds of belief change. Here, *(genuine) revision* takes place when new information about a static world arrives, whereas *updating* tries to incorporate new information about a (possibly) evolving, changing world [KM91]. *Expansion* simply adds new knowledge to the current beliefs, in case that there are no conflicts between prior and new knowledge [Gär88]. *Focusing* [DP97b] means applying generic knowledge to the evidence present by choosing an appropriate context or reference class. *Contraction* [Gär88] and *erasure* [KM91] are operations inverse to revision and updating, respectively, and deal with the problem of how to "forget" knowledge. In this paper, we will make use of this richness of different operations, but only on a surface level, without going into details. The explanations given above will be enough for understanding the approach to be developed here. An interested reader may follow the mentioned references. For a more general approach to belief revision both in a symbolic and numerical framework, cf. [KI01a]. The revision operator $*$ used above is most properly looked upon as a *revision or updating* operator. We will stick, however, to the term *revision*, and will use it in its general meaning, if not explicitly stated otherwise.

The universe of revision operators is given by

$$\mathit{RevisionOp} = \{\mathit{Update}, \mathit{Revision}, \mathit{Expansion}, \mathit{Contraction}, \mathit{Erasure}, \mathit{Focusing}\}$$

and the general task of revising knowledge is realized by a function

$$\mathit{revise} : \mathit{EpState} \times \mathit{RevisionOp} \times \mathcal{P}(\mathit{Sen}_{\mathcal{Q}}) \to \mathit{EpState}$$

A call $\mathit{revise}(\Psi, \mathit{op}, \mathcal{S})$ yields a new state where Ψ is modified according to the revision operator op and the set of sentences $\mathcal{S}$. Note that we consider here belief revision in a very general and advanced form: We revise epistemic states by sets of conditionals – this exceeds the classical AGM-theory by far which only deals with sets of propositional beliefs.

The constraints the function *revise* is expected to satisfy depend crucially on the kind of revision operator used in it and also on the chosen framework (ordinal or e.g. probabilistic). Therefore, we will merely state quite basic constraints here, which are in accordance with the AGM theory [AGM85] and its generalizations [DP97a,KI01a].

The first and most basic constraint corresponds to the *success postulate* in belief revision theory: if the change operator is one of *Update*, *Revision*, *Expansion*, the new information is expected to be present in the posterior epistemic state:

$$\begin{aligned} \mathit{revise}(\Psi, \mathit{Revision}, \mathcal{S}) &\models_{\mathcal{Q}} \mathcal{S} \\ \mathit{revise}(\Psi, \mathit{Update}, \mathcal{S}) &\models_{\mathcal{Q}} \mathcal{S} \\ \mathit{revise}(\Psi, \mathit{Expansion}, \mathcal{S}) &\models_{\mathcal{Q}} \mathcal{S} \end{aligned}$$

Furthermore, any revision process should satisfy *stability* – if the new information to be incorporated is already represented in the present epistemic state, then no change shall be made:

$$\begin{aligned} &\text{If } \ \Psi \models_{\mathcal{Q}} \mathcal{S} \ \text{ then:} \\ &\qquad \mathit{revise}(\Psi, \mathit{Revision}, \mathcal{S}) = \Psi \\ &\qquad \mathit{revise}(\Psi, \mathit{Update}, \mathcal{S}) = \Psi \\ &\qquad \mathit{revise}(\Psi, \mathit{Expansion}, \mathcal{S}) = \Psi \end{aligned}$$

Similarly, for the deletion of information we get:

$$\begin{aligned} &\text{If not } \ \Psi \models_{\mathcal{Q}} \mathcal{S} \ \text{ then:} \\ &\qquad \mathit{revise}(\Psi, \mathit{Contraction}, \mathcal{S}) = \Psi \\ &\qquad \mathit{revise}(\Psi, \mathit{Erasure}, \mathcal{S}) = \Psi \end{aligned}$$

To establish a connection between revising and retracting operations, one may further impose *recovery constraints*:

$$\begin{aligned} &\mathit{revise}(\mathit{revise}(\Psi, \mathit{Contraction}, \mathcal{S}), \mathit{Revision}, \mathcal{S}) = \Psi \\ &\mathit{revise}(\mathit{revise}(\Psi, \mathit{Erasure}, \mathcal{S}), \mathit{Update}, \mathcal{S}) = \Psi \end{aligned}$$

A correspondence between inductive knowledge representation and belief revison can be established by the condition

$$\mathit{inductive}(S) \ = \ \mathit{revise}(\mathit{uniform}, \mathit{Update}, S). \tag{3}$$

Thus, inductively completing the knowledge given by S can be taken as revising the non-knowledge representing epistemic state *uniform* by updating it to S.

In CondorASM, revision is realized by the rule

if $do = Revision$
then $currstate := revise(currstate, rev\_op, new\_information)$
$do := undef$

where the monitored functions $rev\_op : RevisionOp$ and $new\_information : \mathcal{P}(Sen_\mathcal{Q})$ provide the type of revision operator to be applied and the set of new sentences to be taken into account, respectively.

Example 5. In a probabilistic framework, a powerful tool to revise (more appropriately: update) probability distributions by sets of probabilistic conditionals is provided by the *principle of minimum cross-entropy* which generalizes the principle of maximum entropy in the sense of (3): Given a (prior) distribution, P, and a set, $\mathcal{S}$, of probabilistic conditionals, The *MinEnt-distribution* $P_{ME} = MinEnt(P, \mathcal{S})$ is the (unique) distribution that satisfies all constraints in $\mathcal{S}$ and has minimal cross-entropy with respect to P, i.e. P_{ME} solves the minimization problem

$$\min R(P', P) = \sum_\omega P'(\omega) \log \frac{P'(\omega)}{P(\omega)} \tag{4}$$
$$\text{s.t. } P' \text{ is a probability distribution with } P' \models \mathcal{S}$$

If $\mathcal{S}$ is basically compatible with P (i.e. P-*consistent*, cf. [KI01a]), then P_{ME} is guaranteed to exist (for further information and lots of examples, see [Csi75,PV92,Par94,KI01a]). The cross-entropy between two distributions can be taken as a directed (i.e. asymmetric) information distance [Sho86] between these two distributions. So, following the principle of minimum cross-entropy means to revise the prior epistemic state P in such a way as to obtain a new distribution which satisfies all conditionals in $\mathcal{S}$ and is as close to P as possible. So, the *MinEnt*-principle yields a probabilistic belief revision operator, $*_{ME}$, associating to each probability distribution P and each P-consistent set $\mathcal{S}$ of probabilistic conditionals a revised distribution $P_{ME} = P *_{ME} \mathcal{S}$ in which $\mathcal{S}$ holds.

Example 6. Suppose that some months later, the agent from Example 3 has changed his mind concerning his formerly held conditional belief (*young*|*student*) – he now believes that *students* are *young* with a probability of 0.9. So an updating operation has to modify P^* appropriately. We use *MinEnt*-revision to compute $P^{**} = revise(P^*, Update, \{(b|a)[0.9]\})$. The result is shown in the table below.

ω	$P^{**}(\omega)$	ω	$P^{**}(\omega)$	ω	$P^{**}(\omega)$	ω	$P^{**}(\omega)$
abc	0.2151	$ab\overline{c}$	0.1939	$a\overline{b}c$	0.0200	$a\overline{b}\overline{c}$	0.0255
$\overline{a}bc$	0.1554	$\overline{a}b\overline{c}$	0.1401	$\overline{a}\overline{b}c$	0.1099	$\overline{a}\overline{b}\overline{c}$	0.1401

It is easily checked that indeed, $P^{**}(b|a) = 0.9$ (only approximately, due to rounding errors).

4.5 Diagnosis

Having introduced these first abstract functions for belief revision, we are already able to introduce additional functions. As an illustration, consider the function

$$\mathit{diagnose}: \mathit{EpState} \times \mathcal{P}(\mathit{Fact}_{\mathcal{Q}}) \times \mathcal{P}(\mathit{SimpleFact}_{\mathcal{U}}) \rightarrow \mathcal{P}(\mathit{SimpleFact}_{\mathcal{Q}})$$

asking about the status of certain simple facts $D \subseteq \mathit{SimpleFact}_{\mathcal{U}} = \Sigma$ in a state Ψ under the condition that some particular factual knowledge $\mathcal{S}$ (so-called evidential knowledge) is given. It is defined by

$$\mathit{diagnose}(\Psi, \mathcal{S}, D) = \mathit{belief}(\mathit{revise}(\Psi, \mathit{Focusing}, \mathcal{S}), D)$$

Thus, making a diagnosis in the light of some given evidence corresponds to what is believed in the state obtained by adapting the current state by focusing on the given evidence.

Diagnosis is realized by the rule

if $\mathit{do} = \mathit{Diagnosis}$
  then $\mathit{believed\_sentences} := \mathit{diagnose}(\mathit{currstate}, \mathit{evidence}, \mathit{diagnoses})$
    $\mathit{do} := \mathit{undef}$

where the monitored functions $\mathit{evidence} : \mathcal{P}(\mathit{Fact}_{\mathcal{Q}})$ and $\mathit{diagnoses} : \mathcal{P}(\mathit{SimpleFact}_{\mathcal{U}})$ provide the factual evidence and a set of (unquantified) facts for which a degree of belief is to be determined.

Example 7. In a probabilistic framework, focusing on a certain evidence is usually done by conditioning the present probability distribution correspondingly. For instance, if there is certain evidence for being a student and being unmarried – i.e. $\mathit{evidence} = \{\mathit{student} \wedge \mathit{unmarried}[1]\}$ – and we ask for the degree of belief of being young – i.e. $\mathit{diagnoses} = \{\mathit{young}\}$ – for $\mathit{currstate} = P^*$ from Example 3, the system computes

$$\mathit{diagnose}(P^*, \{\mathit{student} \wedge \mathit{unmarried}[1]\}, \{\mathit{young}\}) = \{\mathit{young}[0.8270]\}$$

and update *believed_sentences* to this set. Thus, if there is certain evidence for being an unmarried student, then the degree of belief for being young is 0.8270.

4.6 What-If-Analysis: Hypothetical Reasoning

There is a close relationship between belief revision and generalized nonmonotonic reasoning described by

$$\mathcal{R} \mid\!\sim_{\Psi} \mathcal{S} \quad \text{iff} \quad \Psi * \mathcal{R} \models_{\mathcal{Q}} \mathcal{S}$$

(cf. [KI01a]). In this formula, the operator $*$ may be a revision or an update operator. Here, we will use updating as the operation to study default consequences. So, *hypothetical reasoning* carried out by the function

$$\mathit{nmr}_{\mathit{upd}}: \mathit{EpState} \times \mathcal{P}(\mathit{Sen}_{\mathcal{Q}}) \times \mathcal{P}(\mathit{Sen}_{\mathcal{U}}) \rightarrow \mathcal{P}(\mathit{Sen}_{\mathcal{Q}})$$

can be defined by combining the *belief*-function and the *revise*-function:

$$nmr_{upd}(\Psi, \mathcal{S}_q, \mathcal{S}_u) \;=\; belief(revise(\Psi, Update, \mathcal{S}_q), \mathcal{S}_u)$$

The assumptions $\mathcal{S}_q$ used for hypotherical reasoning are being hold in the monitored function $assumptions : \mathcal{P}(Sen_{\mathcal{Q}})$ and the sentences $\mathcal{S}_u$ used as goals for which we ask for the degree of belief are being hold in the monitored function $goals : \mathcal{P}(Sen_{\mathcal{U}})$. Thus, we obtain the rule

if $do = What\text{-}If\text{-}Analysis$
  then $believed\_sentences := nmr_{upd}(currstate, assumptions, goals)$
    $do := undef$

Example 8. With this function, hypothetical reasoning can be done as is illustrated e.g. by "Given P^* in Example 3 as present epistemic state – i.e. $currstate = P^*$ –, what would be the probability of $(b|c)$ – i.e. $goals = \{(b|c)\}$ –, provided that the probability of $(b|a)$ changed to 0.9 – i.e. $assumptions = \{(b|a)[0.9]\}$?" Condor's answer is $believed\_sentences = \{(b|c)[0.7404]\}$ which corresponds to the probability given by P^{**} from Example 6.

4.7 Conditional Knowledge Discovery

Conditional knowledge discovery is modelled by a function

$$CKD : \; EpState \rightarrow \mathcal{P}(Sen_{\mathcal{Q}})$$

that extracts measured facts and rules from a given state Ψ that hold in that state, i.e. $\Psi \models_{\mathcal{Q}} CKD(\Psi)$. More significantly, $CKD(\Psi)$ should be a set of "interesting" facts and rules, satisfying e.g. some minimality requirement. In the ideal case, $CKD(\Psi)$ yields a set of measured sentences that has Ψ as its "designated" representation via an inductive representation formalism *inductive*. Therefore, discovering most relevant relationships in the formal representation of an epistemic state may be taken as solving the *inverse representation problem* (cf. [KI00,BKI02b]):

> Given an epistemic state Ψ find a set of (relevant) sentences S that has Ψ as its designated representation, i.e. such that $inductive(S) = \Psi$.

The intended relationship between the two operations *inductive* and *CKD* can be formalized by the condition

$$inductive(CKD(\Psi)) \;=\; \Psi$$

which holds for all epistemic states Ψ.

This is the theoretical basis for our approach to knowledge discovery. In practice, however, usually the objects knowledge discovery techniques deal with are not epistemic states but statistical data. We presuppose here that these data are at hand as some kind of distribution, e.g. as a frequency distribution or an ordinal distribution (for an approach to obtain possibility distributions from data, cf. [GK97]). These distributions will be of the same type as epistemic states in the corresponding framework, but since they are ontologically different, we prefer to introduce another term for the arguments of *CKD*-functions:

$$\textit{distribution} : \textit{EpState}$$

if *do* = *CKD*
  then *believed_sentences* := *CKD*(*distribution*)
  do := *undef*

For instance, in a quantitative setting, Ψ may be a (rather complex) full probabilistic distribution over a large set of propositional variables. On the other hand, $CKD(\Psi)$ should be a (relatively small) set of probabilistic facts and conditionals that can be used as a faithful representation of the relevant relationships inherent to Ψ, e.g. with respect to the *MaxEnt*-formalism (cf. Section 4.1). So, the inverse representation problem for *inductive* = *MaxEnt* reads like this: Find a set $CKD(\Psi)$ such that Ψ is the uniquely determined probability distribution satisfying $\Psi = MaxEnt(CKD(\Psi))$.

We will illustrate the basic idea of how to solve this *inverse MaxEnt-problem* by continuing Example 3.

Example 9. The probability distribution we are going to investigate is P^* from Example 3. Starting with observing relationships between probabilities like

$$P^*(\overline{a}b\overline{c}) = P^*(\overline{a}\overline{b}\overline{c}),$$

$$\frac{P^*(abc)}{P^*(ab\overline{c})} = \frac{P^*(\overline{a}bc)}{P^*(\overline{a}b\overline{c})},$$

$$\frac{P^*(a\overline{b}c)}{P^*(a\overline{b}\overline{c})} = \frac{P^*(\overline{a}\overline{b}c)}{P^*(\overline{a}\overline{b}\overline{c})},$$

the procedure described in [KI01b] yields the set $\mathcal{S}_u = \{(b|a), (b|c)\}$ of unquantified (structural) conditionals not yet having assigned any probabilities to them. Associating the proper probabilities (which are directly computable from P^*) with these structural conditionals, we obtain

$$\mathcal{S} \;=\; CKD(P^*) \;=\{(b|a)[0.8],\;(b|c)[0.7]\}$$

as a *MaxEnt*-generating set for P^*, i.e. $P^* = MaxEnt(\mathcal{S})$. In other words, the probabilistic conditionals

(*young*|*student*)[0.8],
(*young*|*unmarried*)[0.7]

that have been generated from P^* fully automatically, constitute a consise set of uncertain rules that faithfully represent the complete distribution P^* in an information-theoretically optimal way. So indeed, we arrived at the same set of conditionals we used to build up P^* in Example 3. Thus, in this case we have

$$CKD(MaxEnt(\mathcal{S})) \;=\; \mathcal{S}$$

But note, that in general, $\mathit{CKD}(P)$ will also contain redundant rules so that only

$$\mathit{CKD}(\mathit{MaxEnt}(\mathcal{S})) \supseteq \mathcal{S}$$

will hold.

5 Conclusions and Further Work

Starting from a bird's-eye view of the CONDOR system, currently being under construction within our CONDOR project, we developed a high-level ASM specification for a system that provides powerful methods and tools for managing knowledge represented by conditionals. Thereby, we were able to elaborate crucial interdependencies between different aspects of knowledge representation, knowledge discovery, and belief revision.

Whereas in this paper, we deliberately left the universe $\mathcal{Q}$ of quantitative and qualitative scales abstract, aiming at a broad applicability of our approach, in a further development step we will refine CONDORASM by distinguishing quantitative logic (such as probabilistic logic) and qualitative approaches (like ordinal conditional functions). In vertical refinement steps, we will elaborate the up to now still abstract functions like *belief* and *revise* by realizing them on lower-level data structures, following the ASM idea of stepwise refinement down to executable code.

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A Appendix: Universes, functions, constraints, and transition rules for CondorASM

A.1 Universes

universe	typical element	
Σ	a	simple fact
Ω	ω	complete conjunction over Σ
$\mathcal{Q}$	x	measure, degree of belief
$SimpleFact_{\mathcal{U}} = \Sigma$	a	(unquantified) simple fact
$Fact_{\mathcal{U}}$	A	(unquantified) fact
$Rule_{\mathcal{U}}$	$(B\|A)$	(unquantified) conditional, rule
$Sen_{\mathcal{U}} = Fact_{\mathcal{U}} \cup Rule_{\mathcal{U}}$		(unquantified) sentences
$SimpleFact_{\mathcal{Q}}$	$a[x]$	quantified simple fact
$Fact_{\mathcal{Q}}$	$A[x]$	quantified fact
$Rule_{\mathcal{Q}}$	$(B\|A)[x]$	quantified conditional, rule
$Sen_{\mathcal{Q}} = Fact_{\mathcal{Q}} \cup Rule_{\mathcal{Q}}$		quantified sentences
$EpState$	Ψ	epistemic state
$WhatToDo$		domain of actions to be performed
$RevisionOp$	op	domain of revision operators

A.2 Static functions

$$\models_{\mathcal{Q}} \,:\, EpState \times Sen_{\mathcal{Q}} \rightarrow Bool$$

$$uniform : EpState$$

$$inductive : \mathcal{P}(Sen_{\mathcal{Q}}) \rightarrow EpState$$

$$belief : EpState \times \mathcal{P}(Sen_{\mathcal{U}}) \rightarrow \mathcal{P}(Sen_{\mathcal{Q}})$$

$$revise : EpState \times RevisionOp \times \mathcal{P}(Sen_{\mathcal{Q}}) \rightarrow EpState$$

$$diagnose : EpState \times \mathcal{P}(Fact_{\mathcal{Q}}) \times \mathcal{P}(\Sigma) \rightarrow \mathcal{P}(Sen_{\mathcal{Q}})$$

$$nmr_{upd} : EpState \times \mathcal{P}(Sen_{\mathcal{Q}}) \times \mathcal{P}(Sen_{\mathcal{U}}) \rightarrow \mathcal{P}(Sen_{\mathcal{Q}})$$

$$CKD : EpState \rightarrow \mathcal{P}(Sen_{\mathcal{Q}})$$

A.3 Dynamic functions

1. **Controlled functions**:

function	arity	
$currstate$	$: EpState$	current epistemic state
$believed\_sentences$	$: \mathcal{P}(Sen_{\mathcal{Q}})$	believed sentences w.r.t. current action

2. **Monitored functions**:

function	arity	
rule_base	: $\mathcal{P}(Sen_{\mathcal{Q}})$	set of rules for initialization
stored_state	: *EpState*	epistemic state for loading
queries	: $\mathcal{P}(Sen_{\mathcal{U}})$	unquantified sentences for querying
new_information	: $\mathcal{P}(Sen_{\mathcal{Q}})$	set of new rules for revision
rev_op	: *RevisionOp*	revison operator
evidence	: $\mathcal{P}(Fact_{\mathcal{Q}})$	evidence for diagnosis
diagnoses	: $\mathcal{P}(SimpleFact_{\mathcal{U}})$	possible diagnoses to be checked
assumptions	: $\mathcal{P}(Sen_{\mathcal{Q}})$	assumptions for hypoth. reasoning
goals	: $\mathcal{P}(Sen_{\mathcal{U}})$	goals for hypothetical reasoning
distribution	: *EpState*	distribution for conditional knowledge discovery

3. **Interaction functions**:

function	arity	
do	: *WhatToDo*	current action to be performed

A.4 Constraints

$$belief(\Psi, \mathcal{S}) = \{S[x] \mid S \in \mathcal{S} \text{ and } \Psi \models_{\mathcal{Q}} S[x]\}$$

$$uniform = inductive(\emptyset)$$

$$revise(\Psi, Revision, \mathcal{S}) \models_{\mathcal{Q}} \mathcal{S}$$
$$revise(\Psi, Update, \mathcal{S}) \models_{\mathcal{Q}} \mathcal{S}$$
$$revise(\Psi, Expansion, \mathcal{S}) \models_{\mathcal{Q}} \mathcal{S}$$

If $\Psi \models_{\mathcal{Q}} \mathcal{S}$ then:

$$revise(\Psi, Revision, \mathcal{S}) = \Psi$$
$$revise(\Psi, Update, \mathcal{S}) = \Psi$$
$$revise(\Psi, Expansion, \mathcal{S}) = \Psi$$

If not $\Psi \models_{\mathcal{Q}} \mathcal{S}$ then:

$$revise(\Psi, Contraction, \mathcal{S}) = \Psi$$
$$revise(\Psi, Erasure, \mathcal{S}) = \Psi$$

$$
\begin{aligned}
\mathit{revise}(\mathit{revise}(\Psi, \mathit{Contraction}, \mathcal{S}), \mathit{Revision}, \mathcal{S}) &= \Psi \\
\mathit{revise}(\mathit{revise}(\Psi, \mathit{Erasure}, \mathcal{S}), \mathit{Update}, \mathcal{S}) &= \Psi
\end{aligned}
$$

$$
\begin{aligned}
\mathit{inductive}(S) &= \mathit{revise}(\mathit{uniform}, \mathit{Update}, S). \\
\mathit{diagnose}(\Psi, \mathcal{S}, D) &= \mathit{belief}(\mathit{revise}(\Psi, \mathit{Focusing}, \mathcal{S}), D) \\
\mathit{nmr}_{\mathit{upd}}(\Psi, \mathcal{S}_q, \mathcal{S}_u) &= \mathit{belief}(\mathit{revise}(\Psi, \mathit{Update}, \mathcal{S}_q), \mathcal{S}_u) \\
\mathit{inductive}(\mathit{CKD}(\Psi)) &= \Psi
\end{aligned}
$$

A.5 Transition rules

if *do* = *Initialization*
then *currstate* := *inductive*(*rule_base*)
do := *undef*

if *do* = *Load*
then *currstate* := *stored_state*
do := *undef*

if *do* = *Query*
then *believed_sentences* := *belief*(*currstate*, *queries*)
do := *undef*

if *do* = *Revision*
then *currstate* := *revise*(*currstate*, *rev_op*, *new_information*)
do := *undef*

if *do* = *Diagnosis*
then *believed_sentences* := *diagnose*(*currstate*, *evidence*, *diagnoses*)
do := *undef*

if *do* = *What-If-Analysis*
then *believed_sentences* := $\mathit{nmr}_{\mathit{upd}}$(*currstate*, *assumptions*, *goals*)
do := *undef*

if *do* = *CKD*
then *believed_sentences* := *CKD*(*distribution*)
do := *undef*